FIRST SEMESTER B.C.A. DEGREE EXAMINATION, JANUARY 2012

(CCSS)

B.C.A.—Complementary Course

CA 1C 01—MATHEMATICAL FOUNDATION FOR COMPUTER APPLICATION

Time: Three Hours

Maximum: 30 Weightage

Part A (Objective Type Questions)

Answer all twelve questions.

1. Define a null set.
2. Let a set contains \( n \) elements, then how many subsets are there?
3. Let A and B be two sets. Define a relation from A to B.
4. Give an example of an odd function.
5. Let A be a square matrix of order \( n \). When we can say that the matrix B is an inverse of A.
6. Define a skew symmetric matrix.

Fill in the blanks:

7. A well-defined collection of objects is known as a ________.
8. Let \( A = \{2, 4, 5, 6, 7\} \), then the total number of its subsets is ________.
9. Two sets A and B are said to be ________ if and only if every element of A is an element of B and consequently every element of B is an element of A.
10. Let A and B be two sets. Then the set \( \{a \in A : (a, b) \in R, \text{ for some } b \in B\} \) is called the ________ of R.
11. A relation \( R \) on a set A is ________ if whenever \( (a, b) \in R \) and \( (b, c) \in R \) then \( (a, c) \in R \).
12. Let \( A(x) = \int_{a}^{x} f(x) \, dx \) for all \( x \geq a \). Then \( A'(x) = ________ \). \( (12 \times \frac{1}{4} = 3 \text{ weightage}) \)

Part B (Short Answer Questions)

Answer all nine questions.

13. List all the subsets of the set \( A = \{a, b, c\} \).
14. Let \( A = \{1, 2, 3, 4\} \), \( B = \{0, 1, 3, 5, 7\} \) and \( C = \{2, 4, 6, 8\} \) then find
   
   (a) \( A \cup B \) 
   (b) \( A \cap B \) 
   (c) \( A - B \) 
   (d) \( B \cup C \).

Turn over
15. Let $A = \{2, 3, 5\}$ and $B = \{6, 8, 10\}$. Define a binary relation $R$ from $A$ to $B$ as follows. For all $(x, y) \in A \times B$, $(x, y) \in R \iff x$ divides $y$. Write $R$ and $R^{-1}$.

16. Differentiate $\frac{7\sin x - 5}{\cos x}$.

17. Integrate $\frac{3x^3 - 5x^2 + 6x}{x}$.

18. Write the identity matrix of order 3.
19. Give an example of a lower triangular matrix of order 3.
20. Define a function from a set $S$ to a set $T$.

21. Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$. Show that $B$ is the inverse of $A$. 

\textbf{Part C (Short Essay Questions)}

Answer any five questions.

22. If the function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 
3x - 1 & \text{if } x > 3 \\
x^2 - 2 & \text{if } -2 \leq x \leq 3 \\
2x + 3 & \text{if } x < -2
\end{cases}$$

find:

(a) $f(2)$.
(b) $f(4)$.
(c) $f(-1)$.
(d) $f(-3)$.

23. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and if $n$ is a positive integer, find $n$.

24. Write the product rule and hence evaluate the derivative of $x^3 \sin x$.

25. Find $\int e^{\sec x} \sec x \tan x \, dx$. 
26. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) for the function \( 2 \sin x + \cos x \).

27. Evaluate \( \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^3 x \, dx \).

28. Find \( A \) \( (B + C) \) when
   \[
   A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.
   \]

\( (5 \times 2 = 10 \text{ weightage}) \)

**Part D (Essay Questions)**

*Answer any two questions.*

29. (a) Write the quotient rule for differentiation.

   (b) Differentiate \( \frac{e^x}{1 + \sin x} \).

   (c) Using quotient rule find the derivative of
       
       (i) \( \tan x \).
       
       (ii) \( \sec x \).

30. (a) Differentiate:

   (i) \( x^2 e^x \sin x \).

   (ii) \( (x \sin x)^3 \).

   (b) Integrate:

   (i) \( \int \frac{x^2 + 1}{x^2 - 5x + 6} \, dx \).

   (ii) \( \int x \log x \, dx \).

31. (a) Find \( x, y, z \) and \( t \) if

   \[
   2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 3 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}.
   \]

   (b) Find \( A \) and \( B \) if \( A + B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \) and \( A - B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \).

\( (2 \times 4 = 8 \text{ weightage}) \)